APPLICATION OF LATTICE GAS TECHNIQUES TO THE STUDY OF SEDIMENT EROSION AND TRANSPORT CAUSED BY LAMINAR SHEETFLOW

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ABSTRACT

In order to gain a deeper understanding of the dynamics of erosion and sediment transport on hillslopes, it seems important to clarify the role of some basic mechanisms involved in these processes. While there is evidence that this cannot be done using the theoretical framework of river hydraulics, the use of numerical analysis could be of considerable help. The nature of the problem requires a technique capable of solving Navier–Stokes equations at low Reynolds number, with geometrically complex boundaries and solid particles moving inside the flow field. All these requirements make a novel method, known as lattice gas automaton LGA, a natural candidate for the study of the hydrodynamics of sheetflows. However, due to the recent introduction of this technique, there is a lack of a clear definition of its operational limits. Considering the case of a viscous sheetflow on an erodible rough boundary, we argue that by using LGA the stream Reynolds number can be increased only at the expense of a reduction of the boundary shearing stress. Accordingly, LGA cannot profitably be used to study the beginning of sediment motion and transport. On the other hand, a further evolution of LGA, known as the lattice Boltzmann method, seems highly promising for the numerical study of the erosion processes that eventually lead to drainage network evolution along hillslopes. © 1997 by John Wiley & Sons, Ltd.

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INTRODUCTION

It is widely recognized that a correct understanding and quantification of the mechanisms leading to erosion and transport on hillslopes is a crucial issue, both from theoretical and practical points of view. For the geomorphologist, these processes are among the key factors which control landscape evolution; for agricultural engineers, an understanding of the processes governing rill and inter-rill erosion is of primary importance to prevent crop land loss and to choose between different agricultural practices. Furthermore, hydraulic and environmental engineers are interested in erosion processes since they control the suspended sediment load transported by rivers.

In spite of its great importance, this issue seems to be neglected in the literature if we compare the number of papers concerning the mechanics of sediment transport on hillslopes with the great number of contributions about erosion and transport in rivers. In recent decades, however, a considerable effort has been devoted to fill this gap, mostly by means of field and laboratory studies.

Nevertheless, sediment transport mechanics is still mainly analysed within the framework of river hydraulics (e.g. Roth *et al*., 1989). Consider, for instance, the erosion and transport processes caused by sheetflow on a cohesionless soil. Apart from the role of rainfall in detaching soil particles, there is evidence that the mechanics of sediment erosion and transport is quite different from that typical of rivers. The first situation is dominated by hydraulically smooth and laminar flows, the latter by rough, turbulent flows. These different hydrodynamic regimes give rise to different structures of the flow field and, consequently, of the fluid dynamic

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force field around sediment particles of the bed. The equilibrium of a single grain must therefore be analysed by means of different approaches.

As an example, Yalin (1977, 1992) identifies the condition of particle entrapment in rough turbulent regime as:

$$L/W > 1 \tag{1}$$

where L (in N) is the lifting force acting on the grain and W (in N) is its submerged weight. This relation, based on the direct observation of the processes arising at the beginning of sediment motion, clearly implies that the grain detachment is due entirely to the action of the lift. Most of the currently used physically based bedload formulas, such as those developed by Einstein (1950), Yalin (1963) and Bagnold (1973), have been derived on this interpretative basis.

However, according to the experimental work of Coleman (1967), Watters and Rao (1971) and of Davies and Samad (1978), there is evidence that the lift, in the hydraulic conditions prevailing in sheetflows, is bedward oriented. This would imply that the use of Relation 1 in discussing sheetflow erosion is inappropriate, because the role of the lift, if any, is stabilizing rather than mobilizing, and the onset of the motion of grains is governed by drag force. Accordingly, the motion of detatched grains is rolling rather than saltating in laminar sheetflow, and the use of transport equations specifically derived for rough turbulent conditions to model the transport capacity for overland flow would be questionable, at least at a conceptual level. This is the case of the Yalin (1963) equation, suggested by Neibling and Foster (1980) for use as part of the CREAMS model (Knisel, 1980), and of the Einstein equation used by Roth *et al*. (1989) in their hillslope evolution model. It is relevant to note here that Francis (1973) has shown, by analysing multi-exposure photographs of grain trajectories moving on a rough fixed bed of a laminar stream, that bedload transport by saltation can sometimes occur in the absence of fluid turbulence. Nevertheless, as far as the micromechanics of sediment particles is concerned, this similitude reflects only the kinematics of the process but not its dynamics. As a matter of fact, while in the case of turbulent motion, particle saltation occurs mainly as a consequence of the disruption of the viscous sublayer by burst evolution that can induce positive lift forces, in the case considered by Francis it is purely a ballistic effect. In other words, particle entrainment does not occur by saltation in a laminar flow.

We believe that an interesting contribution to this and other debated questions on the mechanics of sediment motion in the field of inter-rill erosion could come by the numerical solution of the flow field around a rough, movable boundary. This task is considerably hindered by the complexity of the boundary geometry, the biphase nature of the solid–fluid flow and the presence of a free surface, potentially perturbed by the momentum transfer of incoming raindrops. In this paper we investigate this problem and seek a solution by means of a novel numerical technique.

THE LATTICE GAS AUTOMATON AS A NUMERICAL METHOD FOR THE STUDY OF SOIL EROSION

As part of a wider research programme on sheetflow erosion and transport, we have explored the possibility of gaining further insight on the mechanics of the interaction of laminar sheetflows and incoherent sediments by numerically solving the flow field on an erodible rough boundary using a lattice gas automaton LGA technique.

A discussion of this numerical technique goes beyond the scope of the present contribution; excellent accounts have been proposed by Wolfram (1986) and Frisch *et al*. (1987). In this context, it is relevant to note that the convergence of the constitutive equations of this method to the Navier–Stokes equations has been theoretically demonstrated and numerically verified (Kadanoff *et al*., 1987; d'Humières and Lallemand, 1987) for some well-known hydrodynamic cases. The method is basically an evolution of the kinetic theory of gases. Accordingly, it is not surprising that simulations of incompressible flows are limited by a constraint on the Mach number, Ma, (Schlichting, 1979):

$$Ma = \frac{U}{c} < 0{\cdot}3 \tag{2}$$

where U (m s^{-1}) is the average flow velocity and c (m s^{-1}) is the speed of sound in the fluid. The speed of sound in LGA has been numerically and theoretically investigated (e.g. Frisch *et al*., 1987). Unfortunately, while in real gases its value is sufficiently high to make the constraint of Equation 2 easily satisfied in many applications, in LGA its value is considerably smaller, so imposing a severe limitation on the maximum attainable value of U.

Several lattice gas models have been developed. They basically differ in the number of particles and the collision rules at each lattice site. For our simulations, an FHP III (e.g. Frisch *et al*., 1987) algorithm has been implemented and tested. This model includes one rest particle and implements all the possible collisions at a node preserving mass and momentum. It can be shown that this allows the minimization of the fluid kinematic viscosity. From a theoretical point of view, there are some good reasons to use a LGA to simulate the laminar sheetflow on a moving boundary. Apart from their computational efficiency that allows massive parallel implementation, these methods are particularly appropriate to study viscous flows at low and moderate Reynolds numbers along complex boundaries, as demonstrated by their most promising field of application, that is the small-scale study of the flow fields inside porous media. In addition, these methods have been efficiently extended to simulate flow problems with free boundaries (Clavin *et al*., 1988) and solid fluid suspensions (Ladd *et al*., 1988). All these properties make the LGA a natural candidate for the solution of viscous biphase flow fields.

The study of sediment transport mechanics implies the study of the distribution of stresses on the bed. Accordingly, in order to test our implementation of the FHP-III LGA model, we have accomplished a preliminary evaluation of the drag exerted by a uniform flow on a circular cylinder of infinite extension. The results, obtained numerically by integrating the stress on the cylinder surface, have been compared with the theoretical and experimental values available in the literature. When the computation domain is not sufficiently wide with respect to the typical linear dimension of the cylinder, i.e. its diameter d (m), a systematic overestimation of the theoretical drag values can be observed; this is a well known physical and numerical effect. Provided that the above-mentioned condition is satisfied, the results seen in Figure 1 for a set of simulations in the range $0{\cdot}5<Ud/\nu<30$, (where d (m) is the diameter of the cylinder and ν (m^2 s^{-1}) the fluid kinematic viscosity) are satisfactory and show the possibility of using the LGA method to compute the hydrodynamic force exerted on a body immersed in a moving stream.

In spite of these encouraging results, the evaluation of the drag exerted by a laminar stream on the grains of an erodible bed during some preliminary simulations has shown that its value decreases when the stream

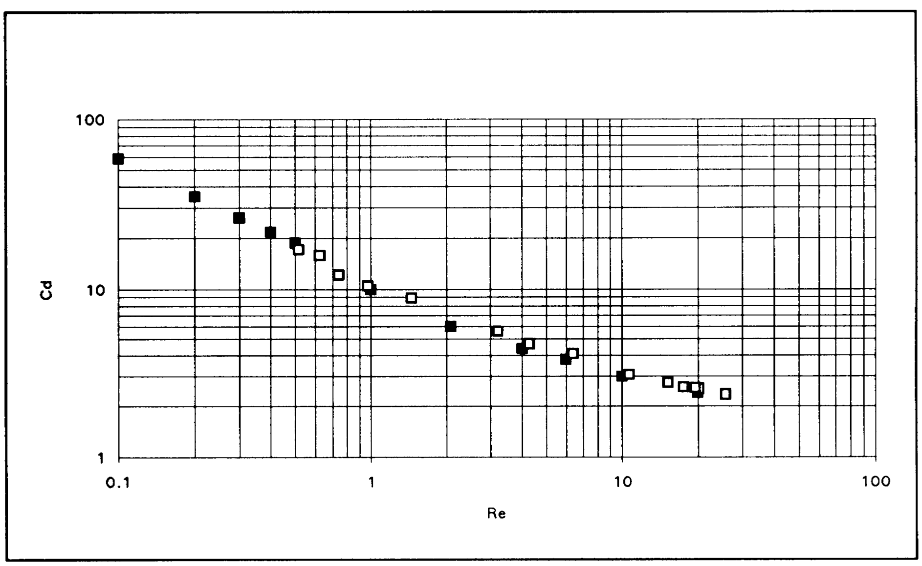

Figure 1. Drag coefficient, C_d, for circular cylinders as a function of the Reynolds number, Ud/ν. The open squares correspond to the values obtained by solving the flow field with the *LGA* method, and the solid squares to experimental data.

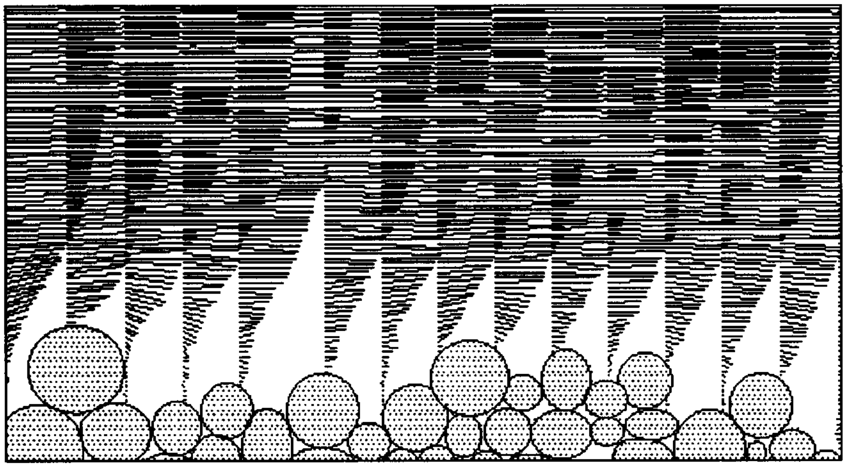

Figure 2. Bidimensional laminar flow field on a rough boundary computed by a *FHP-III LGA*. The vertical dimension has been discretized in 226 lattice units. The scale of the velocity profiles has been magnified, the maximum velocity being about 0·3 lattice unit per time step, in order to respect the incompressibility constraint (Equation 2). Accordingly, velocity gradients along the lower rough boundary are flatter.

Reynolds number is increased. Accordingly, besides the well known incompressibility constraint which limits the maximum attainable velocity in a LGA simulation, we have found a further constraint in the method, previously unknown in the literature. This constraint reduces the applicability range of the method in the field of sediment erosion and transport.

IDENTIFICATION OF A LIMIT OF THE LGA METHOD

In order to discuss this restriction, let us consider the incompressible inner shear flow between two parallel, rough and indefinite plates. This situation can be regarded as an idealization of the free surface flow field over an erodible bed before the inception of erosion (see Figure 2). For the sake of simplicity, we shall consider a single-phase flow and we shall demonstrate that the LGA technique is not suitable for the study of the onset of the two-phase transport phenomenon.

The two-dimensional stationary and uniform motion can be fully defined by the set of six characteristic parameters ρ, μ, D, v_*, h, g, where ρ (kg m^{-3}) is the fluid mass density, μ (Pa) is its dynamic viscosity, D (m) is the linear dimension which characterizes the wall roughness, v_* (m s^{-1}) is the shear velocity, h (m) is the flow depth and g (m s^{-2}) is the acceleration due to gravity.

If we select from the set above the three variables ρ, v_*, h as basic quantities, by applying the procedures of dimensional analysis (Sedov, 1982), we can easily derive a group of three possible dimensionless variables governing the phenomenon:

$$X_1 = \frac{hv_*}{\nu} \tag{3}$$

$$X_2 = \frac{h}{D} = Z \tag{4}$$

$$X_3 = \frac{v_*^2}{hg} \tag{5}$$

It can be easily shown that X_3 is the friction slope J. If τ_0 (Pa) is the shear stress on the bed surface, then, by definition:

$$\tau_0 = \gamma h J = \rho v_*^2 \tag{6}$$

and hence

$$X_3 = \frac{\rho v_*^2}{h\gamma} = J$$

Similarly, the dimensionless variable X_1 reflecting the influence of the kinematic viscosity v, can be expressed in terms of the flow Reynolds number Re:

$$Re = \frac{4Uh}{\nu}$$

where U (m s^{-1}) is the average velocity of the flow. Considering the laminar flow inside a rough, rectangular duct, the average velocity U can be defined by the Darcy–Weisbach relation:

$$J = \lambda(Z,Re)\,\frac{U^2}{8gh} = \frac{96f(Z)}{Re}\,\frac{U^2}{8gh} \tag{7}$$

The friction coefficient λ is defined as:

$$\lambda = 96f(Z)/Re$$

where $f(Z)$ is a function of the relative roughness Z only, whose value tends to 1 as Z is greater than 10. Rewriting λ in terms of the flow and friction velocities, we can easily write:

$$\frac{U}{v_\star} = \sqrt{\frac{8}{\lambda}} = \sqrt{\frac{8Re}{96f(Z)}}$$

and, as a consequence, X_1 can be redefined as:

$$X_1 = \frac{hv_\star}{v} = \frac{4hU}{v}\,\frac{v_\star}{4U} = Re\,\frac{1}{4}\sqrt{\frac{96f(Z)}{8Re}} = \sqrt{Re}\sqrt{\frac{3}{4}f(Z)} \tag{8}$$

Accordingly, every dimensional function of the six variables mentioned above can be replaced either by a dimensionless function of the three parameters:

$$X_1, X_2, X_3 \equiv X_1, Z, J$$

or, equivalently, by a function of

$$Re, Z, J \tag{9}$$

Let us consider, as an example, the dimensionless expression of the bed shear stress τ_0. It can be written either as:

$$\bar{\varphi}_{\tau_0} = \frac{\tau_0}{\rho v_\star^2} = 1$$

or, equivalently, in the form:

$$\bar{\varphi}_{\tau_0} = \frac{\tau_0}{\rho U^2} = \frac{1}{Re} f(Z) \tag{10}$$

For convenience, henceforth we shall use the set of dimensionless variables (Set 9) with reference to Equation 10 for the dimensionless shear stress.

During the experimental determination of the critical shear stress for an erodible bed, after the characteristic diameter of the grains has been chosen, the space of the dimensionless variables (Set 9) is usually explored by varying the fluid inflow rate. Obviously, Re, Z and J cannot be varied independently, because of the constraint:

$$f(Re, Z, J) = 0 \tag{11}$$

the exact form of which depends on the flow regime. In the case of laminar flow, as considered above, the relation 11 can be easily represented in explicit form as:

$$\frac{JgD^3}{\nu^2} = \frac{3}{4} f(Z) \frac{Re}{Z^3} \tag{12}$$

Once the absolute roughness D and the fluid have been chosen, Equation 12 can be represented as a surface in the space of the variables (Set 9), and it can be further examined during the experiments in the laboratory.

Let us now consider the same problem, from the numerical perspective, using an LGA technique. We could be tempted to believe that the path just outlined can be followed numerically. Unfortunately, as we found out, this is not the case. Because of the incompressibility constraint, when using this method the flow Reynolds number can be increased only by enlarging the characteristic linear dimension h. This is because the other two variables that can increase it are already constrained by a fixed value. The kinematic viscosity ν is fixed, depending on the set of collisions of the chosen FHP model. On the other hand, the average velocity U has a very limited range of variation owing to the restriction on the Mach number, and is usually maximized in order to reduce the white noise that, in this method, affects the results (Wolfram, 1986). Consequently, Re can be increased only by acting on the relative roughness Z, according to the relationship:

$$Re = CZ \tag{13}$$

where C is a constant, and the set of attainable simulations is only that deriving from the system:

$$\begin{cases} \dfrac{JgD^3}{\nu^2} = \dfrac{3}{4} f(Z) \dfrac{Re}{Z^3} \\ Re = CZ \end{cases} \tag{14}$$

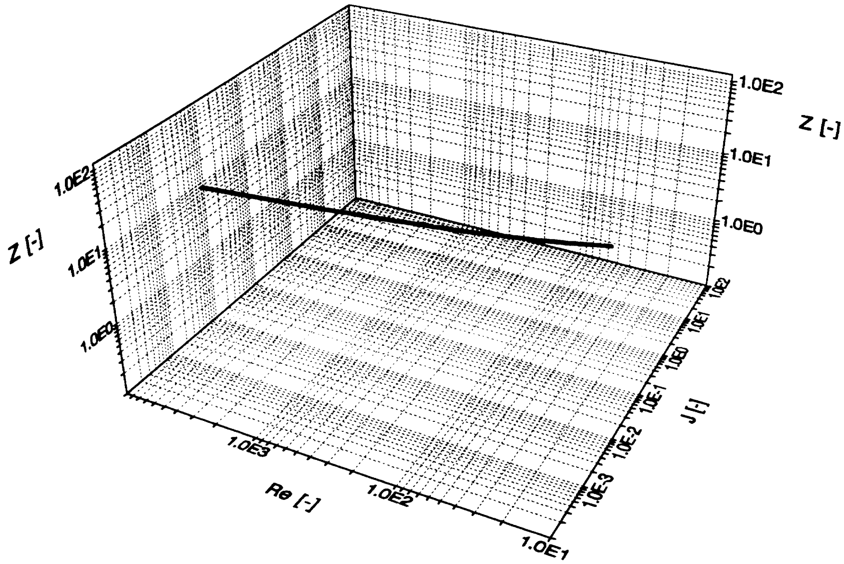

Figure 3. The dimensionless parameters' space. Each point inside it corresponds to a specific flow condition, identified by the value of the three adimensional groups *Re*, the stream Reynolds number, *J*, the slope friction, and *Z*, the relative roughness. The pathline is the subset in Equation 14 containing all the possible experiments which can be performed with an LGA model when the maximum flow velocity *U* is constrained.

Therefore, only some of the potential combinations of *Re, Z* and *J* implicit in Equations 11 and 12 can be investigated.

In particular, in the case of the viscous flow considered in this paper, the solution of the system in Equation 14 is the line shown in Figure 3, in the space of the variables (*X,Z,J*). It is observed that the rise in the Reynolds number is accompanied by a rise in *Z* and a simultaneous decrease of the slope friction *J*. In particular, the maximum value of *J* that can be simulated depends on the minimum value of *Z*. This implies, independently of the setting of the grains of the bottom, the existence of a constraint on the maximum attainable value of the bed shear stress. As an example, in the case $Z>10$, Equation 10 can be rewritten as:

$$\varphi_{\tau_0} = \frac{\tau_0}{\rho U^2} = \frac{12}{Re} \tag{15}$$

Therefore, after *U* has been maximized, the bed shear stress τ_0 is a decreasing function of *Re*. Clearly, this does not lead to a simultaneous increase in *Re* and τ_0 as required in a numerical simulation of the inception of sediment motion and transport.

FUTURE DEVELOPMENTS

In spite of the advantages (e.g. memory efficiency, intrinsic parallelism and stability, ability to model complicated moving boundaries and biphase flows) that make the LGA method ideally adapted to the numerical simulation of sheetflow and rillflow erosion processes, the existence of the limit identified above greatly restricts the scope of LGA in geomorphological problems, helping to clarify which calculations can actually be done with LGA. As shown above, the identified constraint basically stems from the very limited dynamics of the viscosity coefficient in LGA simulations. Another problem of simulations with LGA is the considerable amount of statistical fluctuation that afflicts the results. As a consequence, the hydrodynamic information can be derived only after a computationally expensive ensemble averaging of the original data provided by the method.

On the other hand, the discrete space, time and velocity equation:

$$f_\alpha(X+e_\alpha,\ T+1)=f_\alpha(X,\ T)+\Omega_\alpha \tag{16}$$

that in LGA simulations governs the dynamics of the quantity:

$$f_\alpha(X,T) \qquad (\alpha=1,\ldots,6) \tag{17}$$

(where $f_\alpha(X,T)$ is a Boolean variable that indicates the presence at the lattice site X and time T of a particle with velocity in the direction α on the hexagonal lattice used to discretize two-dimensional flow fields, and Ω_α is a collision operator) can be replaced by a similar equation in term of the one-particle velocity distribution function $n_\alpha(X,T)$, where n_α is a continuous quantity. The latter equation can be derived from the Boltzmann equation of kinetic theory (e.g. Cercignani, 1969), that describes the dynamics of the density $n(v,x,t)$ of particles with a given velocity u at a given space–time point (x,t). It has been suggested (McNamara and Zanetti, 1988; Higuera and Jimenez, 1989) that the equivalent of Equation 16 which can be derived in this way, provides a convenient substitution to LGA and this novel numerical method has been called the lattice Boltzmann equation (LBA). This is essentially the kinetic equation resulting from ensemble-averaging of the FHP cellular automaton dynamics. This method, instead of following the microscopic evolution of individual molecules as in LGA, deals with one-particle distribution functions. In so doing, it replaces the Boolean variables that represent the presence or absence of particles at a site by the real-valued mean populations, that is, fluctuation-free, while it retains most of the advantages previously discussed for LGA.

Here, it is important to observe that the transport coefficients (e.g. the fluid viscosity) arising from the two methods are different. In particular, while in LGA v is minimized by optimizing the collision rules between particles, in LBA the same result can be obtained by the tuning of a few parameters in the collision operator, in a more general way, independently of any collision rule. Accordingly, the viscosity coefficient can be decreased to a considerably larger extent than in LGA, and the subspace in the *Z–Re–J* space of Figure 3 can be significantly widened, as will be explored in future research.

CONCLUSIONS

In this contribution,the applicability of the lattice gas technique LGA to solving Navier–Stokes equations in relation to problems of sheetflow erosion and transport is discussed. The method has been selected for its ability to cope in an efficient way with complex geometries, with moving boundaries like those induced by the presence of a free surface potentially perturbed by raindrops, and with two-phase flows like those following the inception of grains inside an eroding stream flow. An LGA FHP-III model has been implemented and numerically tested, verifying the convergence of the results to some analytical solutions of the Navier–Stokes equations and to some experimental results for the drag of a circular cylinder. In spite of these encouraging results, considering the case of viscous sheetflow on an erodible rough boundary, we show that, by using this method, the stream Reynolds number can be increased only at the expense of a reduction of the boundary shear stress. This drawback limits the range of applicability of LGA techniques to the study of erosion, and suggests the opportunity for continuing research of a numerical method suitable for the solution of the hydrodynamics of sheetflows and rillflows by using lattice Boltzmann methods.

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